ALL SAINTS COLLEGE LIBRARY TRIVANDRUM

(Pages : 3)

G – 1464

Reg. No. : .....

Name : .....

Sixth Semester B.Sc. Degree Examination, April 2019
First Degree Programme Under CBCSS
Mathematics
Core Course – XI
MM 1643 – COMPLEX ANALYSIS – II
(2014 Admn. Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first **10** questions are **compulsory**. **Each** carries **1** mark.

1. Write the power series expansion of $f(z)=\frac{1}{1+z}$ in the disc $|z|<1$.
2. Test whether the series $\sum_{n=0}^{\infty}\frac{z^n}{n!}$ is convergent in C.
3. What are the singular points of $ze^{\frac{1}{z}}$?
4. Define essential singularity for a function f(z) at z = a.
5. Describe the nature of singularity for $f(z)=\frac{1-e^{2z}}{z^4}$ at z = 0.
6. What is the order of the pole for $f(z)=\frac{1}{(2\sin z-1)^2}$?
7. Find the residue of $f(z)=\frac{e^z}{z^2}$ at its singularity.
8. What is the residue of $f(z)=\frac{1}{z}+1+z+z^2+\ldots$ at z = 0 ?
9. What is the order of the zero of $z(e^z-1)$?
10. If f is an even function, then what is the value of $\int_{-\infty}^{\infty} f(x)dx$?

P.T.O.

SECTION – II

Answer **any 8** questions from this Section. **Each** question carries **2** marks.

11. Find a power series expansion for $f(z) = \frac{z-1}{z+1}$ about $z = 0$.

12. Prove that $\int_C \frac{\sin z}{\left(z - \frac{\pi}{2}\right)^2} dz$, where C is the circle $|z| = 2$.

13. Evaluate $\int_C \frac{e^{iz}}{z} dz$ where C is $|z| = 2$.

14. Determine and identify the singularities of $\frac{z}{1+z^2}$.

15. Determine the order of the pole and residue at $z = 0$ for $\frac{\sinh z}{z^4}$.

16. Find the residue of $\cot z$ at $z = 0$.

17. Write the principal part of the function $f(z) = z \exp\left(\frac{1}{z}\right)$ at its isolated singular point and determine the value of the singularity.

18. If a is a zero of order r for $\frac{1}{f(z)}$, then prove that a is a pole of order r for f(z).

19. Find the residue of $\frac{1}{\left(z^2 + a^2\right)^2}$ at $z = ai$.

20. State Jordan's Lemma.

21. Find the residue of $\frac{ze^z}{(z-1)^3}$ at its pole.

22. Find $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

SECTION – III

Answer **any six** questions from this Section. **Each** question carries **4** marks.

23. State and prove Cauchy's theorem.

24. Evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z| = 2$.

25. Evaluate $\int_C \frac{e^z}{(z+2)(z+1)^2} dz$ where C is $|z| = 3$.

26. Let f be analytic inside and on a simple closed curve C. Then prove that

$f'(z) = \frac{1}{2\pi i}\int_C \frac{f(\xi)}{(\xi - z)^2}\, d\xi$, where z is any point inside C.

27. Let f be a function which is bounded and analytic throughout a domain $0 < |z - z_0| < \delta$. Then prove that either f is analytic at z_0 or else z_0 is a removable singular point of δ.

28. Prove that an isolated singularity a of f(z) is a pole if and only if $\lim_{z \to a} f(z) = \infty$.

29. Prove that $\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta} = \frac{2\pi}{3}$.

30. Prove that $\int_0^{\pi} \frac{a\,d\theta}{a^2 + \sin^2\theta} = \frac{\pi}{\sqrt{a^2 + 1}}$, $a > 0$.

31. Find $\sum_{n=0}^{\infty} \binom{2n}{n} \frac{1}{5^n}$.

SECTION – IV

Answer **any 2** questions from this Section. **Each** question carries **15** marks.

32. a) State and prove Cauchy's integral formula. **7**

    b) Find the residue of $\frac{e^z}{z^2(z^2 + 9)}$ at its poles. **8**

33. a) State and prove Casorati-Weierstarss theorem. **7**

    b) Prove that $\int_{-\infty}^{\infty} \frac{(x^2 - x + 2)}{x^4 + 10x^2 + 9}\, dx = \frac{5\pi}{12}$. **8**

34. a) Prove that $\int_0^{\infty} \frac{dx}{(x^2 + a^2)^2} = \frac{\pi}{4a^3}$. **8**

    b) Show that $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$. **7**

35. a) Use residue, evaluate $\int_0^{2\pi} \frac{d\theta}{1 + a\sin\theta} = \frac{2\pi}{\sqrt{1 - a^2}}$, $-1 < a < 1$. **8**

    b) Prove that $\int_0^{\infty} \frac{\cos x}{1 + x^2}\, dx = \frac{\pi}{2e}$. **7**
